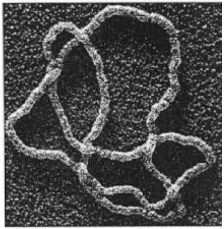
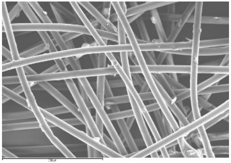
Applied Knot Theory

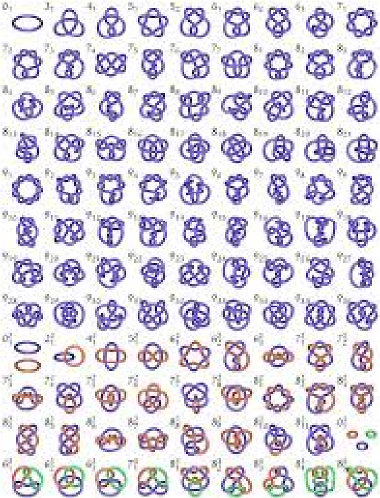
August 17, 2020

Knots and Links





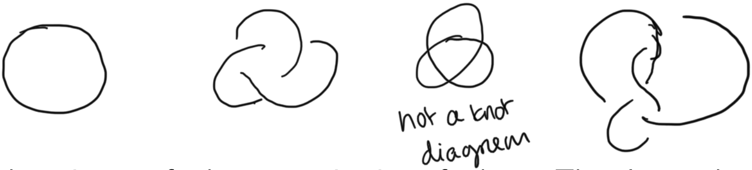
Knots and Links



Knot

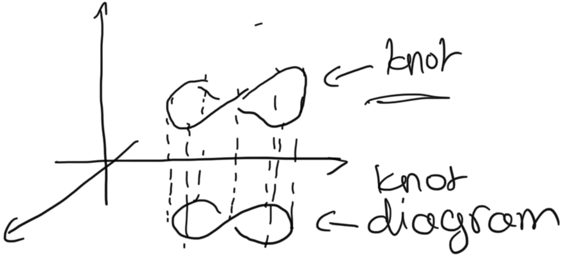
A subset $K \subset R^3$ is a knot if it is the image of a smooth injective map $f : S^1 \to R^3$ with $df/d\theta \neq 0$.
(smooth means that $d^n f/d\theta^n$ exists for all n)

Knot diagrams



We call such a picture of a knot a projection of a knot. The places where the knot crosses itself in the picture are called the crossings of the projection.

A diagram is a planar picture of a knot made up of arcs and crossings



Knot diagrams

If it has only 1 crossing then it is trivial

If a knot is non-trivial has more than one crossing. Work with one crossing to verify.

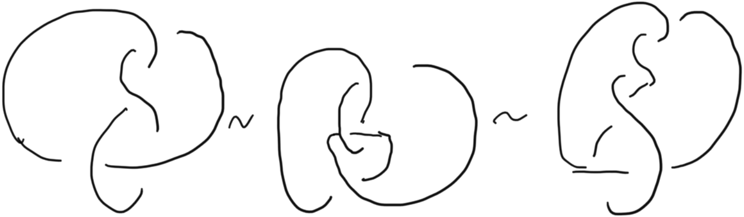
Q: Show that there are no 2-crossing nontrivial knots — if it has only 2 crossings then it is trivial

Case 1: Case 2: Case 3: Case 4

Knot diagrams

these are the trivial knot

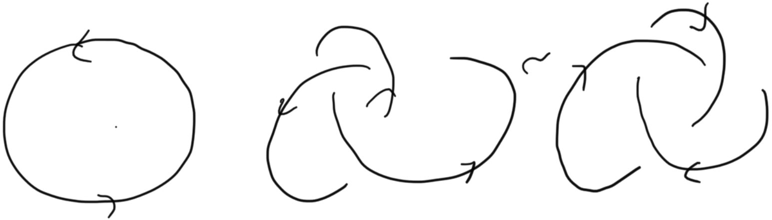
Which knots are different? Which diagrams are different?



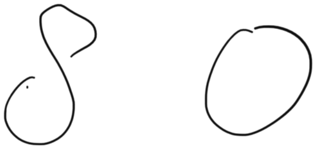
All equivalent! They are the "figure eight knot"

Oriented knots

An orientation is defined by choosing a direction to travel around the knot. This direction is denoted by placing coherently directed arrows along the projection of the knot in the direction of our choice. We then say that the knot is oriented.

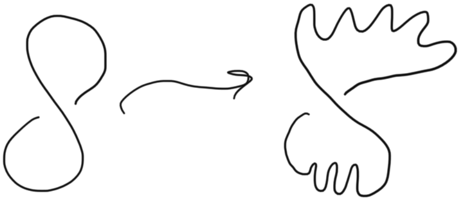


Reidemeister moves



Ambient isotopy: Deformation of one knot to another in 3-space (without crossing through itself)

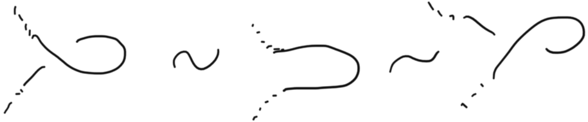
Planar isotopy: Deformation of a diagram preserving the same crossings



Reidemeister moves

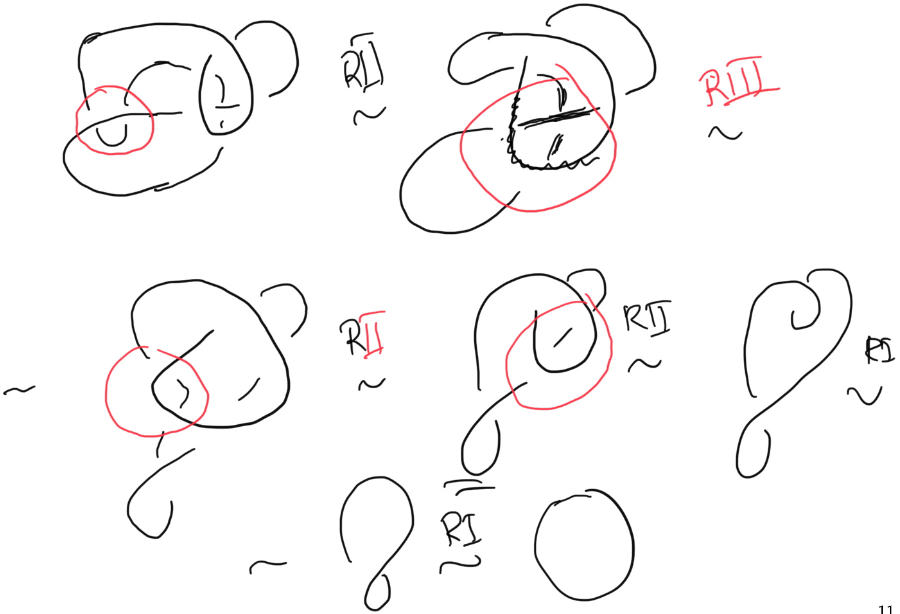
Reidemeister's Theorem: Two knots (links) in space can be deformed to each other by an ambient isotopy if and only if their diagrams can be transformed into each other by planar isotopy and Reidemeister moves.

RI :



RII :

RIII :


RII
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